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Definition of Specific Polynomials Using Generating Functions Other Than Its Applications

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Abstract

It is well knowledge that generating functions are crucial to understanding the theory of traditional orthogonal polynomials. In this paper, we discuss the following problems for systems of polynomials described by generating functions. A differential equation that each polynomial satisfies can be derived in step (A). (B) Discover the general answer to the differential equation that was discovered in (A). (C) Is the general solution found in (B) expressed as a linear combination of functions using generalised hyper geometric functions. This essay aims to demonstrate through two examples how the problem (C) can be successfully resolved.

Keywords: Orthogonal Polynomials, Differential Equation

Introduction:

It is vital to investigate the characteristics of polynomials, and it is both intriguing and important to create new polynomials using new generating functions. Polynomials were first described by Humbert (1921) $\Pi_{n,m}^{\nu}(x)$, n $= 0,1,2, \ldots$, by the generating function

$$(1 - mtx + t^m)^{-\nu} = \sum_{n=0}^{\infty} \Pi_{n,m}^{\nu}(x) t^n. \ (14)$$

Gould (1965) called $\Pi_{n,m}^{\nu}(x)$ the Humbert polynomial of degree n and provided its generalisation. In their study from 1987, Milovanovi and Djordjevic presented a differential equation for the function $\Pi_{n,m}^{\nu}(X)$ using difference operators. The generalized Hermite polynomials were given a definition by Lahiri (1971) $H_{n,m,\nu}(x), n = 0,1,2, \ldots,$ by the generating function

$$\exp(\nu tx - t^m) = \sum_{n=0}^{\infty} H_{n,m,\nu}(x) \frac{t^n}{n!} \ (15)$$

The other generalization of Hermite polynomials by the generating function was presented by Gould and Hopper (1962).

$$x^{-a}(x-t)^a \exp\left(p(x^r - (x-t)^r)\right)$$

The case of a $= 0$ is equivalent to that defined by Bell (1934). We discussed the possibility of defining the polynomials in Suzuki$Q_n(x; k, \nu), n = 0,1,2, \ldots,$ by the use of the following generating function, which has properties that are analogous to those of the Humbert polynomials

$$(1 - 2tx + t^k)^{-\nu} = \sum_{n=0}^{\infty} Q_n(x; k, \nu) t^n,$$

where k is an integer such that k ≥ 2 and ν is a positive real number. Note that

$$\Pi_{n.m}^{\nu}(x) = Q_n(mx/2; m, \nu)$$

and the polynomial $Q_n(x; k, \nu)$ is not entirely new. However, we gave a differential equation for the function $Q_n(x; k, \nu)$ this does not include difference operators and is instead expressed as an explicit expression. Because of this, we were able to derive the general solution to the differential equation at the value of x equal to zero $Q_n(x; k, \nu)$ satisfies. In the research that was published under the title Dobashi (2014), we discussed the possibility of defining a generalization of the Hermite polynomials by the generating function.

$$\exp\left(t^k x - t^{k+j}\right) = \sum_{n=0}^{\infty} R_n(x;k,j)t^n, (16)$$

assuming that k and j are both positive integers. And the findings that we achieved were comparable to those in the instance of $Q_n(x;k,v)$ In this particular instance, the general solution that corresponds to it is written down as a linear combination of functions that are stated by the application of ${}^2F_{k+j-1}$-type hypergeometric functions. The differential equations for are the focus of this paper's research $Q_n(x;k,v)$ and$R_n(x;k,j)$, as well as to obtain the general solutions for them when x = 0 is considered. The discussion about $Q_n(x;k,v)$ is given in, and that for $R_n(x;k,j)$ is given.

Some Polynomials and Special Functions by Using Lie Laplace Transformation

While researching the use of ordinary differential equations in the study of physics in 1953, Courant and Hilbert came upon the concept of special functions. This led to the development of the area of special functions. Also around the same time, Morse and Feshbach were looking at the ways in which special functions may be used to the study of the physical sciences. Therefore, progress was made into the realm of special functions as a result. According to what Paul Turan has seen, the history of special functions goes back a very, very long distance. The majority of the prominent mathematicians who worked in the 18th and 19th centuries, including Euler, Legendre, Laplace, Gauss, Kummer, Riemann, and Ramanujan, made significant contributions to the theory of special functions. Because of their applicability to and interaction with a variety of other subfields within physics and mathematics, such as number theory, combinatorics, computer algebra, and representation theory, the special functions have been the subject of research in the past and continue to do so today. The reader who is interested should check out the wonderful works written by Andrews, Rao, Rose, and Rainville respectively. Miller added to the development of Weisner's theory by establishing a connection between it and the factorization technique, which was developed by Schrodinger. He further extended the theory by establishing a connection between it and Infield and Hull. Kalnins, Onacha, and Miller have done research on Lie algebraic characterizations of two-variable Horn functions. They do this by extending a two-variable Horn function in terms of one-variable hyper geometric functions, which results in the development of a technique for creating generating functions. The hyper geometric functions in one, two, and more variables are discussed throughout the majority of the thesis. These functions are introduced after the definitions and significant features of basic functions such as the Gamma and Beta functions have been presented.

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