



Inventory Model for Deteriorating Substances under Quantity-Dependent Trade Credit with Price-Dependent Demand

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Abstract

This study examines an integrated inventory model under exchange credit when the disintegration rate follows a spectacular circulation. In this case, it is anticipated that the request rate will be included in the selling price, and the request amount will determine how long an installment can be delayed. Deficits in the model are completely amplified. The goal capability to concentrate on the retailer's ideal requesting strategy is taken to be the increase of the absolute benefit per unit of time. The proposed inventory model is used in this paper's helpful application model to support the flexibility of the business. Our findings suggest that when the exchange credit period is broken down, this model can be highly useful in determining the best requesting method.

Keywords: Inventory, Deterioration

1. Introduction

Payments to the provider are made after receiving the items as evident in both deterministic and probabilistic classical inventory models. In time, the supplier will supply the retailer with a deferred term in installments for the purchase measure known as the exchange credit period to generate interest. Giving the merchant such a credit term will increase provider sales and reduce nearby stock levels. However, in the absence of a required installment, the store can profit from a credit term to reduce cost and increase benefit. During the reasonable period, the client is not required to pay interest, but if the installment is delayed past the period, interest will be levied. The client finds this strategy to be very beneficial because he can delay the installment until the end of the allowable deferral term. For the past two decades, the majority of countries have seen a change in their financial woes due to clear Positive Many Meanings - a lessening of the impact of money usage. Money forecasting for the future cannot be disregarded. A number of business experts have created the EOQ design by fusing the effects of your time assessment/estimation of money with a direct time subordinate interest rate. In addition, some researchers have developed a device by taking into account time assessment/estimation of money in a swollen atmosphere. In present section, concern has not been supplied for enhancing little items as grill, pigs, ducks and more. Because of sizable advancement the inventory increments in excess fat and additionally diminish because of death by several maladies. It is the unparalleled to state such inventory model by Hwang in 1999 placed into a stock sort autonomously to improve and rotting things below LIFO and FIFO procedure. It is found an EOQ model for ameliorating and deteriorating items having partial backlogging due to which Influences of Inflation and time value of money are presented. The inadequacies are permitted in the subsequent split up to the time when several shortcomings is accrued, and remainder is missed. Bhole (2014) studied on review on green manufacturing, important, methodology and its application. Aggarwal and Jaggi (2020) in this present research, an inventory type with a summed up exponential diminishing need is perceived as helped through to see the result of parameter switches on the appropriate response. Paul (2001) examined a model where the necessity of things is constrained by both



the present degree of supply and the expense of selling. Vidal (2004) presented a model which permits clients to add an incentive for clients to an organization by joining monetary, cost and buyer examination during its structure procedure, considering an estimation system for characterizing product esteem with natural thought.

2. Mathematical solutions

Now here, mathematical model is built to calculate optimal refill-cycle time which maximizes total annual income of cumulative degrading goods in an inventory system, including short-term payments, depending on the quantity. The inventory volume at time 0 is 'a' and at time t is T. The stock amount is decreased both by demand and depletion during period [0, t₁] and eventually to zero at time t= t₁, inventory levels reduces. Subsequently, shortcomings are permitted, and all demands are completely backlogged throughout [t₁, T].

Like above, an inventory status differential equation is provided

$$\frac{dI(t)}{dt} = \begin{cases} -(a-p) - \theta I(t) & \text{if } 0 \leq t \leq t_1 \\ -(a-p) & \text{if } t_1 \leq t \leq T \end{cases} \quad (1)$$

For BC, I(t) = 0 at t = t₁.

Solving equation (1), we get

$$I(t) = \begin{cases} \frac{(a-p)}{\theta} [e^{\theta(t_1-t)} - 1] & \text{if } 0 \leq t \leq t_1, \\ (a-p)(t_1 - t) & \text{if } t_1 \leq t \leq T \end{cases} \quad (2)$$

The beginning inventory level S for every cycle is obtained as

$$S = I(0) \frac{(a-p)}{\theta} (e^{\theta t_1} - 1) \quad (3)$$

Total number of items D_T that become deteriorated in interval [0, t₁], say D_T, are given by

$$D_T = S - \int_0^{t_1} (a-p) dt = \frac{(a-p)}{\theta} (e^{\theta t_1} - \theta t_1 - 1) \quad (4)$$

So, value of order quantity Q per cycle is obtained to be as under

$$Q = D_T + \int_0^T (a-p) dt = \frac{(a-p)}{\theta} (e^{\theta t_1} - \theta t_1 - 1) + (a-p)T \quad (5)$$

Next, total profit per unit of time of inventory system using various components is as under

(1) Ordering cost equal to A; (2) Holding cost per cycle is

$$Hc = h \int_0^{t_1} I(t) dt = (h(a-p)/\theta^2) (e^{\theta t_1} - \theta t_1 - 1) \quad (6)$$

(3) Purchase cost is

$$CQ = (C(a-p)/\theta) (e^{\theta t_1} - \theta t_1 - 1) + C(a-p)T \quad (7)$$

4) Shortage cost is

$$Cs = C_1 \int_{t_1}^0 I(t) dt = \left(\frac{C_1}{2}\right) (a-p)(T - t_1)^2 \quad (4.24)$$

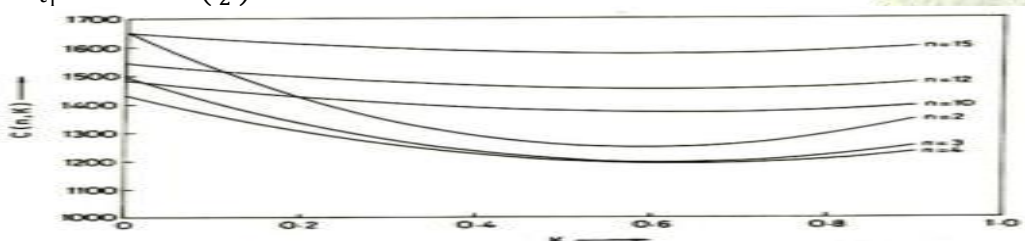


Figure 1: Cost C(n, K) estimation versus K

There are two major cases to happen in interest charged and interest earned during each order cycle and each case is discussed as follows.



Case 1 when $(M \leq t_1)$. Here, length of period with positive stock is larger than credit period, the buyer may use revenue to earn interest at some rate I_e per annum in $(0, t_1)$, which is given by

$$I_e p \int_0^{t_1} -(a - p)(t - t_1) dt = I_e p(a - p)(t_1^2/2) \quad (8)$$

On other hand, after credit period, the unsold stock is financed at interest rate I_k and payable in each order cycle is as under

$$I_k C \int_M^{t_1} (a - p)(t - t_1) dt = (I_e C(a - p)/2)(t_1 - M)^2 \quad (9)$$

Hence,

$$TP_1(t_1, T, p) = p(a - p) - \frac{a-p}{T} \times \left\{ \frac{A}{a-p} + \frac{(h+C\theta)}{\theta^2} (e^{\theta t_1} - \theta t_1 - 1) + CT + \frac{1}{2} [C_1(T - t_1)^2 + I_k C(t_1 - M)^2 - I_e p t_1^2] \right\} \quad (10)$$

Let $t_1 = \gamma T, 0 < \gamma < 1$.

Hence, we get the profit function

$$TP_1(T, p) = p(a - p) - \frac{a-p}{T} \times \left\{ \frac{A}{a-p} + \frac{(h+C\theta)}{\theta^2} (e^{\theta \gamma T} - \theta \gamma T - 1) + CT + \frac{1}{2} [C_1(1 - \gamma)^2 - (I_e p - I_k C)\gamma^2] T^2 + \frac{I_k C}{2} M(M - 2\gamma T) \right\} \quad (11)$$

Since, every organization is to maximize profit function TP_1 , the necessary conditions for which are obtained by conditions of maxima as below

$$\frac{\partial TP_1(T, p)}{\partial T} = a - 2p + \frac{1}{T} \left\{ \frac{(h+C\theta)}{\theta^2} (e^{\theta \gamma T} - \theta \gamma T - 1) - CT - \frac{1}{2} [C_1(1 - \gamma)^2 - (I_e p - I_k C)\gamma^2] T^2 - \frac{I_k C}{2} M(M - 2\gamma T) \right\} = 0 \quad (12)$$

We can calculate optimum values of profit and simultaneously, from equation (11), optimal value of average net profit may be obtained provided and satisfying the conditions

$$\partial^2 TP_1(T, p) / \partial T^2 < 0,$$

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And

$$\left(\frac{\partial^2 TP_1(T, p)}{\partial T^2} \right) \left(\frac{\partial^2 TP_1(T, p)}{\partial p^2} \right) - \left(\frac{\partial^2 TP_1(T, p)}{\partial T \partial p^2} \right) > 0, \text{ at } p = p^*$$

And

$$T = T^*$$

If the solutions obtained from do not satisfy the sufficiency conditions for maxima, then we infer that no feasible solution is optimal for set of parameters taken to solve. Such a situation implies that parameter values are inconsistent and there may be some errors for their estimation

Case 2 when $(M > t_1)$, here buyer pays no interest, but earns interest at some rate I_e for period $[0, M]$. Therefore, the interest earned for in this case is calculated to be

$$(a - p) I_e p t_1 (M - t_1/2)$$

And, total profit per unit time as

$$TP_2(t_1, T, p) = p(a - p) - \frac{a-p}{T} \times \left\{ \frac{A}{a-p} + \frac{(h+C\theta)}{\theta^2} (e^{\theta \gamma T} - \theta \gamma T - 1) + CT + \frac{C_1}{2} - I_e p t_1 \left(M - \frac{t_1}{2} \right) \right\} \quad (13)$$

By time for $\gamma t \leq 1$, the profit function becomes

$$TP_2(T, p) = p(a - p) - \frac{a-p}{T} \times \left\{ \frac{A}{a-p} + \frac{(h+C\theta)}{\theta^2} (e^{\theta \gamma T} - \theta \gamma T - 1) + CT + \frac{C_1}{2} (1 - \gamma)^2 T^2 - \right.$$



$$I_e P \gamma T \left(M - \frac{\gamma T}{2} \right) \quad (14)$$

$\frac{\partial TP_2(T, p)}{\partial p} = 0$ which yield

$$\frac{\partial TP_2(T, p)}{\partial T} = a - 2p + \frac{1}{T} \times \left\{ \frac{(h+C\theta)}{\theta^2} (e^{\theta \gamma T} - \theta \gamma T - 1) + CT + \frac{C_1}{2} (1 - \gamma)^2 - I_e P \gamma T \left(M - \frac{\gamma T}{2} \right) \right\} = 0 \quad (15)$$

If the solutions obtained from do not satisfy sufficient conditions, then we infer that no feasible solution is optimal for set/group of parameters taken/supposed to solve. Such situation implies that parameter values taken are inconsistent and there may be some errors in their estimation. We check/see from the following result, for optimal values of t_1 , t and p

3. Sensitivity Analysis with respect to Parameters

Sensitivity Analysis affectability evaluation is accomplished by changing the parameters from -60% to +60 % (negative and positive) and changing one parameter at time, staying in touch with remainder of the variables at the special attributes of theirs. The affectability examination with different parameters is appeared to the following tables:

Table 1: Sensitivity analysis for demand parameter b

% Change	T^*	t_1^*	Q^*	$Z^*(T^*, t_1^*)$
-40	17.7268	15	2130.06	1910.91
-20	17.7244	15	2133.86	1915.67
0	17.7219	15	2137.66	1920.43
+20	17.7195	15	2141.46	1925.19
+40	17.7171	15	2145.26	1929.95

Table 2: Sensitivity analysis for lifetime parameter θ

%Change	T^*	t_1^*	Q^*	$Z^*(T^*, t_1^*)$
-40	18.9940	15	2431.79	2616.33
-20	18.3755	15	2292.30	2291.17
0	17.7219	15	2137.66	1920.43
+20	17.0623	15	1978.95	1514.85
+40	16.4322	15	1827.27	1094.59

Table 3: Sensitivity analysis for backlogging parameter α

%Change	T^*	t_1^*	Q^*	$Z^*(T^*, t_1^*)$
-40	17.1836	15	2137.66	1947.73
-20	17.4456	15	2137.66	19
0	17.7219	15	2137.66	1920.43
+20	18.0100	15	2137.66	1906.54
+40	18.3084	15	2137.66	1892.67

Table 4: Sensitivity analysis for deterioration parameter θ

%Change	T^*	t_1^*	Q^*	$Z^*(T^*, t_1^*)$
-40	16.7224	15	1887.10	1292.33
-20	17.2138	15	2012.38	1610.94
0	17.7219	15	2137.66	1920.43
+20	18.2478	15	2262.94	2221.03
+40	18.7924	15	2388.22	2512.94



4. Conclusions:

Table 1 reveals that building in requests parameter b relate to increases in the cost of the starting catalogue and the overall price, but it also shows that these builds have a less impact on the length of the inventory cycle. Tables 2 demonstrate how lifespan parameter building relates to decreased starting listing price, overall listing duration, and inventory cycle cost. Table 3 shows that the backlog parameter increases in relation to the length of the inventory cycle, but it also shows that the total cost of inventory decreases as the parameter is evaluated. Table 4 shows that the contained disintegration parameter is expanded and that it is related to the length of the listing cycle, the total cost of the duration, and the inventory cycle. Inventory association must be careful while choosing these system parameters because this model is significantly more sensitive for parameters b, o, and similar values.

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